

Written Exam at the Department of Economics winter 2019-20

Advanced Industrial Organization

Final Exam

19 December 2019

(3-hour closed book exam)

Answers only in English.

This exam question consists of 8 pages in total

Falling ill during the exam

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- contact an invigilator who will show you how to register and submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam.

Be careful not to cheat at exams!

You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

Exercise 1: The Diamond Paradox

N firms produce a perfectly homogenous good at zero marginal cost, $c = 0$. The firms compete in prices, p_1, \dots, p_N (Bertrand), in a one-shot game. There is a unit mass of consumers, each demanding at most one unit of the good. Each consumer knows the price charged by one random store. Consumers can learn the price of other stores by randomly searching one store at a time and learn its price. For each store a consumer searches, a search cost, $s > 0$, is incurred. Consumers have a reserve price $r > 0$ above which demand is zero. That is, consumers will consume the cheapest product they have searched (if multiple firms tie, consumers randomize between them) with a price $p \leq r$. If a consumer has searched only products with $p > r$, they will leave the market without purchasing. Consumers hold an expectation about market prices, p_1^E, \dots, p_N^E which they use to optimally decide how many products to search. In equilibrium (we will be looking for one in pure strategies) we require that 1) the actual prices p_1, \dots, p_N form a Bertrand Nash equilibrium between firms, 2) consumer expectations are correct so $p_i^E = p_i \forall i \in \{1 \dots N\}$ and 3) consumers search and consume optimally given their expectations.

Question 1a

Show that with $s > 0$, there is an equilibrium in which all firms charge $p_1 = p_2 = \dots p_N = r$ (the monopoly price). (Hint: Start by assuming $p_1 = \dots = p_N = r$ is an equilibrium and consider the incentives for each firm to deviate)

Question 1b

Show that the competitive outcome $p_1 = p_2 = \dots p_N = c$, is not an equilibrium with $s > 0$. (Hint: Start by assuming $p_1 = \dots = p_N = c$ is an equilibrium and consider the incentives for each firm to deviate)

Question 1c

Show that the equilibrium in Question 1a is the unique pure strategy equilibrium. (Note: there is no mixed strategy equilibrium in the game. This you don't have to show). (Hint: Start from an arbitrary, potentially asymmetric, pure strategy equilibrium p_1, \dots, p_N and consider the incentives of the lowest price firm)

Exercise 2: A Model of Sales

N firms produce a perfectly homogenous good. The firms compete in prices, p_1, \dots, p_N (Bertrand), in a one-shot game. Firms produce q units at total cost $C(q) = F$ with $F > 0$ being a fixed cost, and with zero marginal costs $C'(q) = 0$. The demand consists of a mass $I \geq 0$ of informed consumers and a mass $M \geq 0$ of uninformed consumers. We let $U = M/N$ denote the mass of uninformed consumers per firm. Each consumer demands at most one unit of the good. There is a reserve price $r > 0$ above which no consumer will purchase. Informed consumers will buy the cheapest product, as long as its price is below r , and uninformed consumers buy a random product, as long the price of this product is below r . If multiple firms tie at the lowest price, each informed consumer will randomize between them such that the tying firms split the informed consumers I equally.

Question 2a

Let p_i be the price of firm i , and p_{-i} a vector of prices of all its competitors. Write up a profit function for firm i , $\pi_i(p_i, p_{-i})$, as a function of p_i and p_{-i} .

Question 2b

Hold the number of firms, N , fixed. Derive an expression for p^* , which is the lowest price any firm is willing to charge. How does p^* depend on N, I and U ?

Question 2c

Construct an argument for why the game does not possess a *symmetric* Nash equilibrium in pure strategies, where all firms charge the same price.

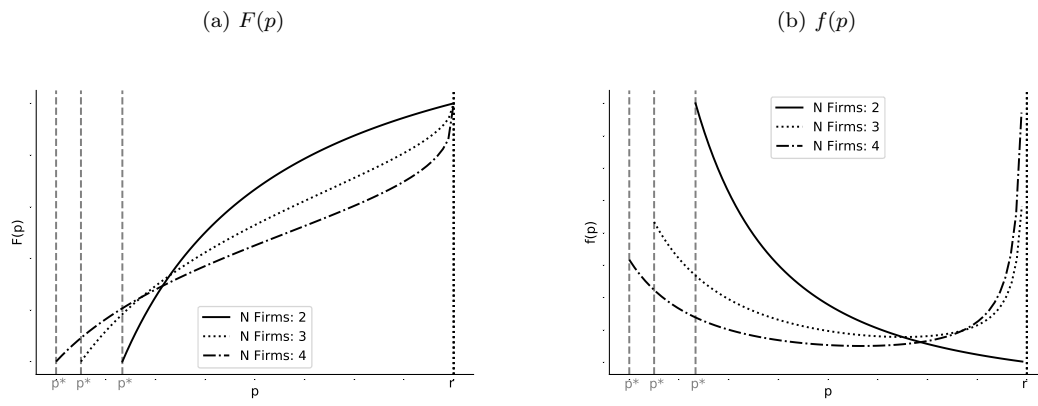
Question 2d

Assuming that the opponents of firm i play each price with a mixing density $f(p)$ over the support $[p^*, r]$, write up the profit maximization problem of firm i , when firm i has to select an optimal mixing distribution. Solve for the symmetric equilibrium mixing cdf, $F(p)$.

Question 2e

Figure 1 (Below) plots the equilibrium pdf, $f(p)$, and the corresponding cdf $F(p)$ for $N = 2, 3, 4$. The lower boundary of the support, p^* is indicated by vertical dashed lines, and the upper boundary of the support, r , is indicated by a vertical dotted line. How does increased competition affect the propensity of firms to submit prices in the bottom, middle and upper range? Provide an intuitive explanation for the relation between competition and the shape of the mixing distribution.

Figure 1: Equilibrium Price Distribution and Number of Firms



Exercise 3

This exercise will ask you questions about the paper “Measuring the Incentive to Collude: The Vitamin Cartels, 1990-1999” by Mitsuru Igami and Takuo Sugaya (WP, 2019). Maintaining the notation from the paper, we consider a set \mathcal{I} of potentially colluding firms participating in repeated Cournot competition over an infinite time horizon, $t = 1, \dots, \infty$. That is, in each period, and for all $i \in \mathcal{I}$, firm i chooses quantity $q_{i,t}$. Firms have differing and time-varying marginal costs, $c_{i,t}$. Firms have a common discount factor $\beta \in (0, 1)$. The firms in \mathcal{I} face a linear inverse demand curve $P_t(Q_{\mathcal{I}}, Q_{fri}) = \tilde{X}_t - \alpha(Q_{\mathcal{I}} + Q_{fri})$ where \tilde{X}_t is a time varying demand shifter, $Q_{\mathcal{I}} = \sum_{i \in \mathcal{I}} q_{i,t}$ is the total output of firms in \mathcal{I} and Q_{fri} is the exogenous output from a set of non-colluding fringe firms. We assume that a collusive agreement amounts to all firms in \mathcal{I} taking Q_{fri} as given, producing a quota $\bar{q}_{i,t}$ such that $\sum_{i \in \mathcal{I}} \bar{q}_{i,t} = \arg \max_Q (P_t(Q, Q_{fri}) - c_*)Q$ where c_* is the marginal cost of the industry leader (that is, the quotas maximize the aggregate profit of firms in \mathcal{I} , assuming they have marginal cost c_*). We assume that firms observe the quantities of each other with a $L = 3$ period lag. We say that non-compliance is confirmed in period τ if some firm did not produce $\bar{q}_{i,t}$ in period $\tau - L$ for the first time. We then suppose that firms agree to play the following equilibrium based on trigger strategies: If no non-compliance is confirmed before period τ then each firm sells $q_{i,\tau} = \bar{q}_{i,\tau}$. If some non-compliance is confirmed in some previous period, $s \leq \tau$ then each firm sells a static Nash equilibrium quantity $q_{i,\tau} = q_{i,\tau}^N$. Lastly, we define the optimal deviation quantity $q_{i,\tau}^D$ which is the best response to all firms in $\mathcal{I} \setminus i$ playing their collusive quotas $\bar{q}_{j \in \mathcal{I} \setminus i, \tau}$. $\bar{q}_{i,t}, q_{i,t}^N$ and $q_{i,t}^D$ gives rise to corresponding stage game profits $\pi_{i,t}^C, \pi_{i,t}^N$ and $\pi_{i,t}^D$ such that $\pi_{i,t}^D > \pi_{i,t}^C > \pi_{i,t}^N$. The paper structurally estimates the Incentive Compatibility Condition (ICC) for collusion to be sustainable in a market, as written in Equation (1)

$$\min_{i \in \mathcal{I}, \tau \geq t} \left(V_{i,\tau|t}^C - V_{i,\tau|t}^D \right) \geq 0 \quad (1)$$

Where $V_{i,\tau|t}^C$ and $V_{i,\tau|t}^D$ measures the expected net present value to firm i of respectively colluding and deviating in period $\tau \geq t$ with the expectation taken in period t and where the minimum is over the set of firms participating in the cartel, \mathcal{I} , and over all future periods $\tau \geq t$. Furthermore, $V_{i,\tau|t}^C$ and $V_{i,\tau|t}^D$ are defined as follows.

$$V_{i,\tau|t}^C = \sum_{s \geq \tau} \beta^{s-\tau} \pi_{i,s|t}^C \quad (2)$$

$$V_{i,\tau|t}^D = \sum_{s=\tau}^{\tau+2} \beta^{s-\tau} \pi_{i,s|t}^D + \sum_{s \geq \tau+3} \beta^{s-\tau} \pi_{i,s|t}^N \quad (3)$$

Question 3a

Provide an explanation for why the condition in Equation (1) is a necessary condition for collusion to be sustainable in a market.

Question 3b

The authors assume that firms can only observe the past actions of each other with a 3-period lag. Based on Equation (1), (2) and (3), please explain how an information lag affects the incentive to collude.

Question 3c

Figure 7 (below) plots empirical estimates of the collective (market) and firm-specific ICC for the Vitamin C market. Figure 8 (below) plots the corresponding ICC for the market leader Roche in the Vitamin A, E, and Beta Carotene market. Based on Figure 7 and 8, please explain the following

- In what year does the estimated ICCs predict that a industry-level cartel is sustainable and not sustainable respectively? Why?
- How does the model prediction of a potential breakdown in the different markets fit reality in the four markets? (The Vitamin C cartel collapsed in 1995 while the others remained in effect until 1999).

Figure 9 (below) plots simulations of the ICC for the Vitamin C cartel under various counterfactual scenarios

- Based on Figure 9, give your analysis of the likely cause of the Vitamin C cartel breakdown.

Question 3d

The paper considers the effect of a hypothetical merger in 1991 between BASF and Takeda (BASF did in fact acquire Takeda in 2001, after the cartel collapse). Because the firms had different marginal costs (Takeda had superior production capability relative to BASF) the authors must make an assumption about how the ex post marginal costs relate to the marginal costs of the merging firms. The authors consider the following relation

$$c_{basf,t}^{post} = (1 - \sigma) \times \min \left\{ c_{takeda}^{pre}, c_{basf}^{pre} \right\} = (1 - \sigma) \times c_{takeda}^{pre}$$

Where $\sigma \in (0, 1)$ is a so-called “efficiency gain” parameter

- Provide an interpretation of the above relation. Do you find this plausible?
- Based on Figure 10 (below), give your analysis of the effect of a merger on cartel stability. How do cost synergies impact the effect of a merger? What is the explanation for this?

Figure 7: Collective and Individual ICCs (Vitamin C)

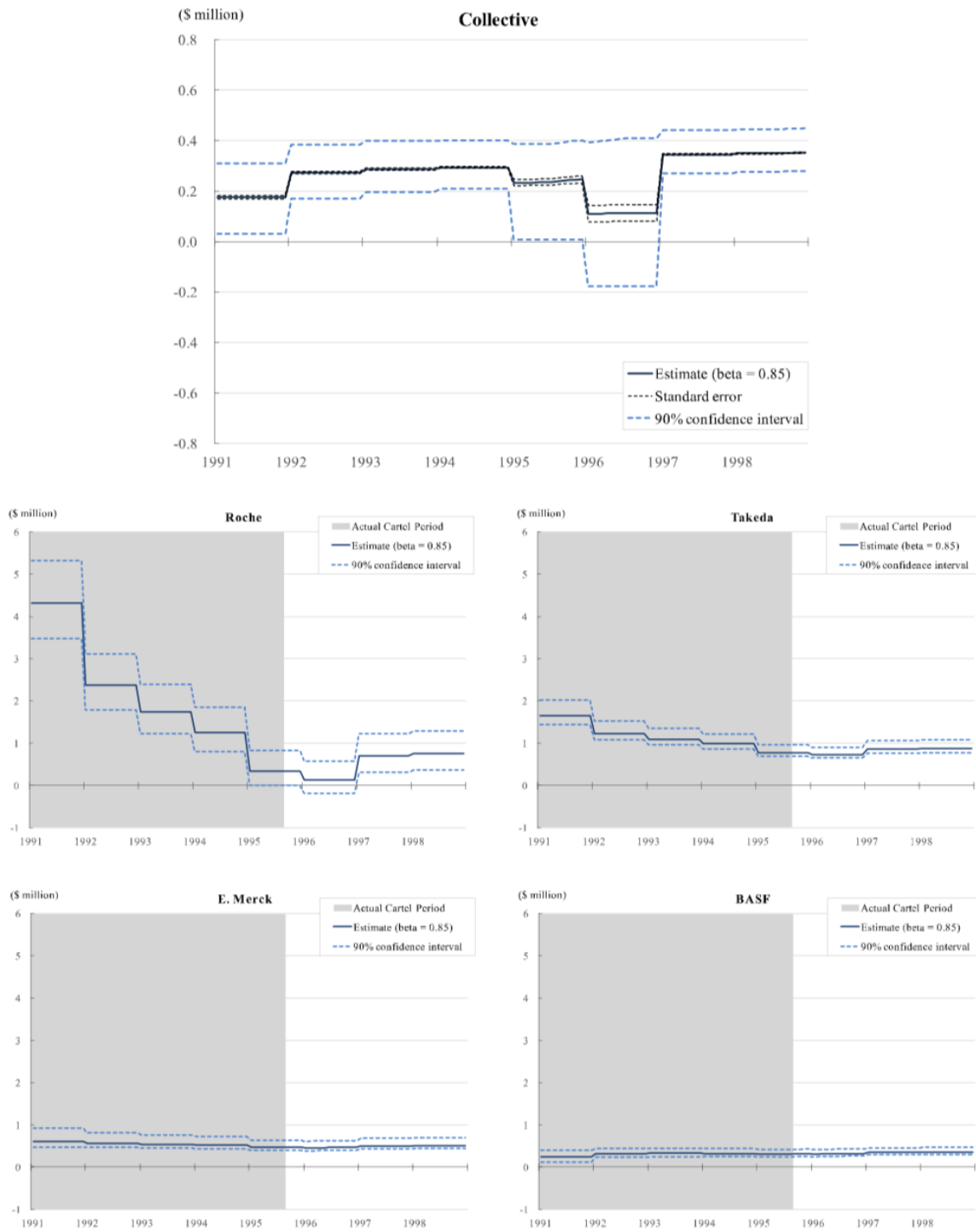


Figure 8: Roche's ICCs (Vitamins A and E, and Beta Carotene)

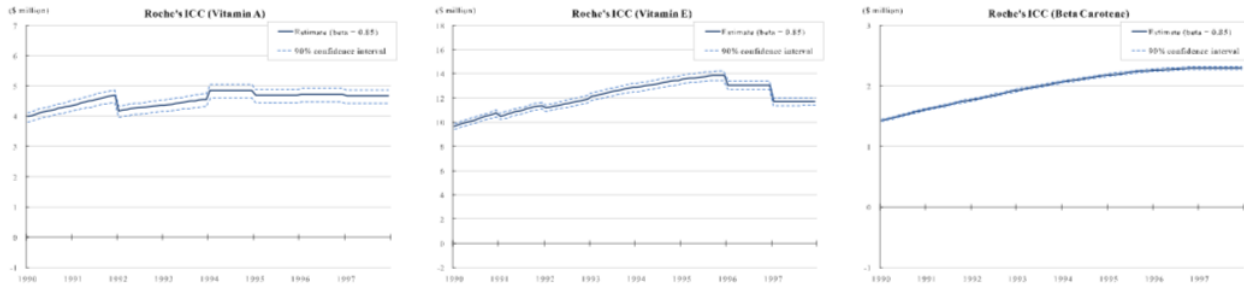


Figure 9: Effects of Demand and Supply on Roche's ICC (Vitamin C)

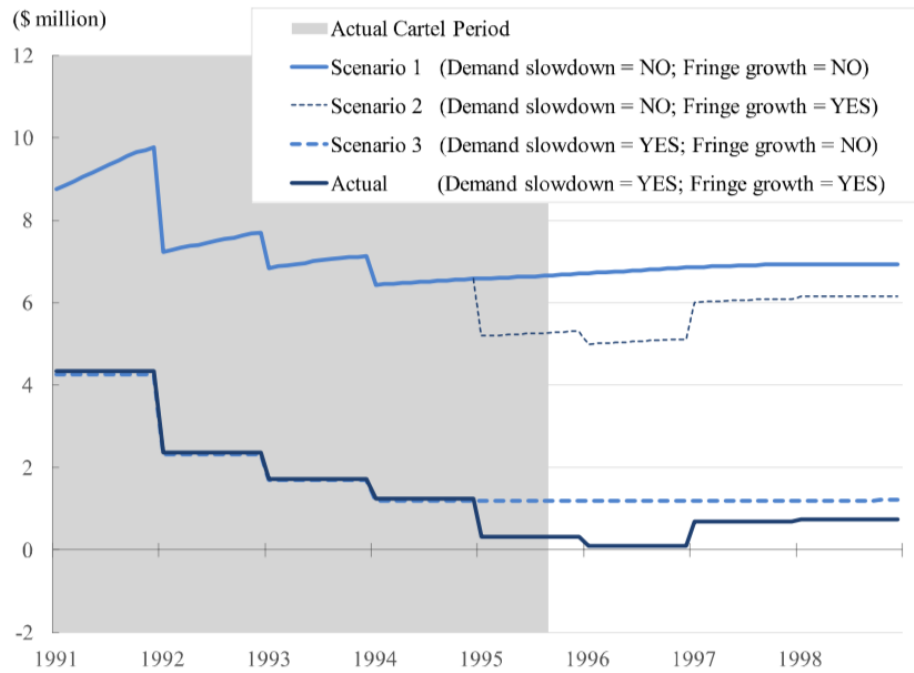
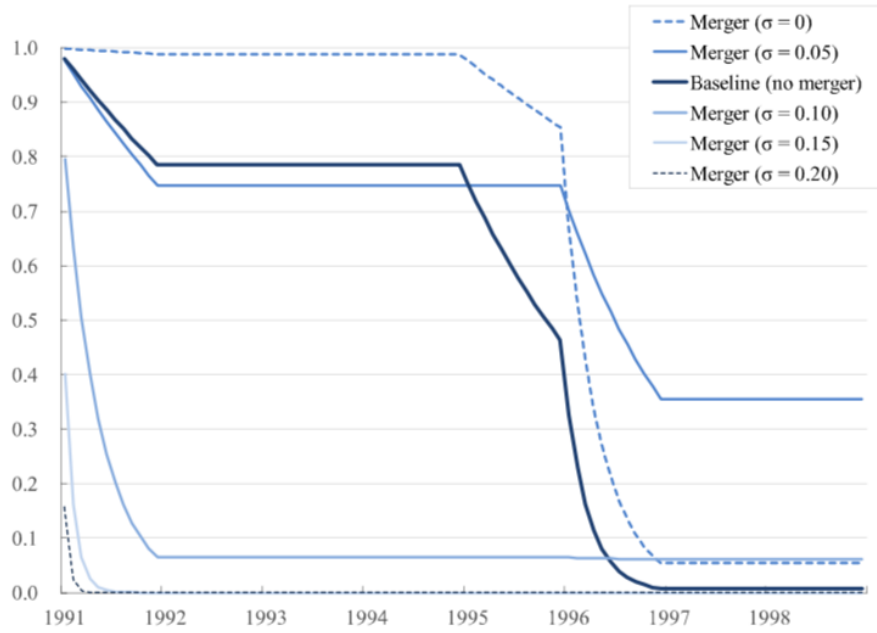


Figure 10: If BASF Had Acquired Takeda's Vitamin C Assets before 1991



Note: Each graph in Figure 10 shows on the y-axis the probability (computed from block bootstrap simulations) that the cartel would have survived beyond a given date, under the assumptions given in the legend.